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ABSTRACT

There is a significant gap between theories of general psychological functions on one hand (e.g., memory) and theories of mathematical content knowledge on the other (e.g., content of algebra). To better guide the design of ground breaking and demonstrably better mathematics instruction, we need instructional principles and associated design methods to fill this gap in a way that is not only consistent with psychological and content theories but prompts and guides us beyond what those theories can do. Toward this goal, I reflect on lessons from past and current Cognitive Tutor mathematics projects. From this experience, I have abstracted four instructional bridging principles, Situation-Abstraction, Action-Generalization, Visual-Verbal, and Conceptual-Procedural, and associated methods for applying them. I illustrate these in the context of the design of the successful Cognitive Tutor Algebra course (now in more than 800 schools) and the on-going research and development of a Cognitive Tutor course for 6th grade mathematics. (Author)

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TOWARD EVIDENCE FOR INSTRUCTIONAL DESIGN PRINCIPLES: EXAMPLES FROM COGNITIVE TUTOR MATH 6

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There is a significant gap between theories of general psychological functions on one hand (e.g., memory) and theories of mathematical content knowledge on the other (e.g., content of algebra). To better guide the design of ground breaking and demonstrably better mathematics instruction, we need instructional principles and associated design methods to fill this gap in a way that is not only consistent with psychological and content theories but prompts and guides us beyond what those theories can do. Toward this goal, I reflect on lessons from past and current Cognitive Tutor mathematics projects. From this experience, I have abstracted four instructional bridging principles, Situation-Abstraction, Action-Generalization, Visual-Verbal, and Conceptual-Procedural, and associated methods for applying them. I illustrate these in the context of the design of the successful Cognitive Tutor Algebra course (now in more than 800 schools) and the on-going research and development of a Cognitive Tutor course for 6th grade mathematics.

Introduction

My first goal in this paper is to summarize a successful educational innovation grounded in psychological research on mathematical learners and learning. Cognitive Tutor Algebra is a full year course that combines a problem-based consumable textbook, used 3 days a week, and intelligent tutoring software, used 2 days a week. Cognitive Tutor Algebra has been successful commercially, sold to over 800 schools, and pedagogically, shown to yield higher student achievement than alternatives. My second goal is to summarize aspects of the psychological theory, ACT-R, that supported the design of this course and helps explain its apparent success. My third goal is to motivate the need for theory at an intermediate level between general cognitive theories, like ACT-R, and models specific to narrow content areas, like proportional reasoning. Such theory is needed to fill the gap that exists between general theory and instructional innovations, so that fewer unguided, purely intuitive decisions need to be made in instructional design. I will discuss four principles to guide such decisions and describe domain-specific methods to implement these principles. My fourth and final goal is to introduce a new Cognitive Tutor mathematics course for 6th grade and to illustrate the four principles using text and tutor lessons from the on-going course development.

Success of Cognitive Tutors and Humility Message

Before discussing the Cognitive Tutor Algebra project, let me start with some background. Following an experience creating and field testing an intelligent tutoring

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system for geometry proof (Koedinger & Anderson, 1993), in which I was one of four teachers involved, it became particularly clear to me how difficult and important it is to attend to the social context of an educational innovation (cf., Lehrer, Randle, & Sancillo, 1989; Schofield, Evans-Rhodes, & Huber, 1990). Two key features of the social context are integrating the innovation with other aspects of the curriculum and innovation-specific teacher professional development. When we started the algebra project in 1992, we took what we called a "client-centered approach" and sought guidance from mathematics educators toward meeting national standards (Koedinger, Anderson, Hadley, & Mark, 1997). Perhaps most importantly, we decided that to address the curriculum integration challenge, we would create a full course including both the text materials (used 3 days a week) and the technology (used 2 days a week). The mathematics curriculum supervisor for the Pittsburgh Public Schools, Diane Briars, pointed us to teacher Bill Hadley who had already been writing new algebra text particularly focused on helping students make sense of algebra. We worked together, combining classroom intuition and cognitive science, to produce a problem-based course that connects multiple representations of functions and employs advanced computational tools. Problems and projects in both the text and software connect to real-world uses of algebraic reasoning like estimating the cost of a rental car, choosing between long-distance phone services, predicting the decline of the condor population, planning profits for shoveling snow, comparing the current quantity and growth rate of old growth forest in the US to the harvest rate.

In three full-year multi-school field studies, we demonstrated that students in a Cognitive Tutor Algebra course learn more than students in comparison classes both on assessments of problem solving and representation use and on standardized assessments of basic skills (Koedinger et al., 1997; Koedinger, Corbett, Ritter, & Shapiro, 2000). As the dissemination of the algebra course was ramping up, in 1995 we obtained funding from local foundations (Heinz, Buhl, Grable, Mellon, and Pittsburgh Foundations) to create full courses for geometry (led by myself) and algebra 2 (led by Albert Corbett). With the help of Carnegie Mellon's technology transfer office, in 1998 we created Carnegie Learning, Inc to further develop and market all three courses. The number of schools using Cognitive Tutor Algebra has roughly doubled each year and is now over 800.

Such success is encouraging, but the job is not done. Education is a very hard problem. Besides tough issues of values, culture, race, politics, respect for teachers, testing, etc., we certainly have not cracked how the mind works or how learning is best achieved. We need to *remain humble*. We need to recognize that our instructional beliefs, practices, theories may be wrong. We need to put them to the test and be proud of failure. In the context of hypotheses and principles, failure advances theory. Given the difficulty of the education problem, we also need to collaborate. No one can successfully tackle a significant educational problem alone. We need to combine expertise

from multiple domains, particularly education researchers, cognitive psychologists and learning scientists, but also anthropologists, computer scientists, measurement, policy experts, etc.

In the remainder of this paper, I reflect upon the lessons learned in our Cognitive Tutor projects particularly at the level of principles and design methods. I illustrate these primarily with examples from the on-going research and development of the Cognitive Tutor course for 6th grade mathematics. In 1999, with support from Carnegie Learning, we began a project to create full Cognitive Tutor courses for middle school mathematics with Albert Corbett leading the 7th and 8th grade course development and myself leading the 6th grade course development.

Theory, Principles, and Methods behind the Design of Cognitive Tutors

ACT-R and General Principles for Instructional Design

Cognitive Tutors are based on the ACT-R theory of learning and performance (Anderson & Lebiere, 1998). The theory distinguishes between tacit performance knowledge, so-called "procedural knowledge" and static verbalizable knowledge, so-called "declarative knowledge". According to ACT-R, performance knowledge can only be learned *by doing*, not by listening or watching. In other words, it is induced from constructive experiences—it cannot be directly placed in our heads. Such performance knowledge is represented in the notation of if-then production rules that associate internal goals and/or external perceptual cues with new internal goals and/or external actions. Three examples of English versions of production rules are shown in Table 1, which is discussed below.

Developing Cognitive Tutor software involves the use of the ACT-R theory and empirical studies of learners to create a "cognitive model". A cognitive model uses a production system to represent the multiple strategies students might employ as well as their typical student misconceptions. The following provides a simplified example from algebra equation where these three production rules are alternative ways to respond to the same goal (a, b, c, and d are any numbers and x is any variable to be solved for):

Strategy 1:	IF the goal is to solve $a(bx+c) = d$ THEN rewrite this as $bx + c = d/a$
Strategy 2:	IF the goal is to solve $a(bx+c) = d$ THEN rewrite this as $abx + ac = d$
Misconception:	IF the goal is to solve $a(bx+c) = d$ THEN rewrite this as $abx + c = d$

The cognitive model is used with an algorithm called model tracing to follow students through their individual approach a problem. In so doing, the tutor is able to provide context-sensitive instruction. The cognitive model is also used by an algorithm called knowledge tracing that assesses students' knowledge growth as they succeed or

fail on actions associated with the production rules in the cognitive model. The results of knowledge tracing are displayed with "skill bars" in the computer tutor interface (see the top right in Figure 5) and are used to select activities and adapt pacing to individual student needs. The cognitive model is not only the key workhouse within the automated Cognitive Tutors, but it is also a theoretical tool used to guide the design of other aspects of instruction including problem and interface design, text materials, and classroom activities.

In 1995, we published a report on the status of the lessons learned to date from Cognitive Tutor development (Anderson, Corbett, Koedinger, & Pelletier, 1995). We described some general Cognitive Tutor design principles consistent with ACT-R and our research and development experience to that date. A key principle recommended to "represent student competence as a production set". In other words, to base instruction on an analysis not of mathematical content per se, but of the way in which students think about the content. The notion of "theorems in action" (Nunes, Schliemann, & Carraher, 1993; Vergnaud, 1982) is very similar to production rules.

The idea here is to characterize how students may think about mathematics differently ("informally" or "intuitively", Resnick, 1987) than is normatively taught or present in textbooks (e.g., see production #1 in Table 1). In addition, production rules represent not just *how* students think about mathematics operations, but *when* they retrieve relevant knowledge. This feature of production rules is critical to representing the fact that the knowledge students acquire is sometimes overly specific, so it does not transfer well, and is sometimes overly general, so it leads to errors. In other words, students' "theorems in action" are often more specific than actual theorems or rules. For instance, students can combine like terms in an equation when coefficients are present (e.g., $2x + 3x \Rightarrow 5x$) but not when a coefficient is missing (e.g., $x - 0.2x$). See production #2 in Table 1. Alternatively, students' theorems in action are often more general than actual theorems or rules. For instance, they may learn to combine numbers by the operator between them (e.g., $2 * 3 + 4 = x \Rightarrow 6 + 4 = x$) without acquiring knowledge that prevents order of operations errors (e.g., $x * 3 + 4 = 10 \Rightarrow x * 7 = 10$). See production #3 in Table 1. It should be clear that the "rules of mathematics" and the "rules of mathematical thinking" are not the same.

A fundamental assumption of ACT-R is that people learn by doing as the brain generalizes from one's explicit and implicit interpretations or "encodings" of one's experiences. This assumption is, in my opinion, quite consistent with constructivism. It is not the information or even the instructional activities students are given per se that matters, but how students experience and engage in such information and activities that determines what knowledge they construct from them. Thus, another principle in Anderson et al. (1995) was "provide instruction in the problem-solving context".

The strength of ACT-R and the principles summarized in Anderson et al. (1995) are that they are general and can apply in multiple domains. However, this strength of generality comes with a limitation. An instructional designer can easily apply the

Table 1. Example Production Rules

Production Rules in English	Example of its application
1. Production acquired implicitly (not explicitly taught) IF you want to find Unknown and the inal result is Known-Result and the last step was to apply Last-Operator to Last-Num, THEN work backwards by inverting Last-Operator and applying it to Known-Result and Last-Num	Part of informal "unwind" strategy that solves problems of the form "3x + 48 = 63" without by working backwards: 63 – 48 -> 15 / 3 = 5
2. Overly specific production IF "ax + bx" appears in an expression and c = a + b THEN replace it with "cx"	Works for "2x + 3x" but not for "x + 3x"
3. Overly general production IF "Num1 + Num2" appears in an expression THEN replace it with the sum	Leads to order of operations error: "x * 3 + 4" is rewritten as "x * 7"

theory and principles in multiple ways, some good, some OK, and some bad. For instance, the principle "minimize working memory load" can be implemented in many ways including having the technology perform trivial operations for students. However, this approach will only be effective if the designer is right about what is trivial. Getting the application of the theory and principles right depends on getting domain-specific details right. *Thus, a general theme of this paper is that we not only need to make progress in better articulating theory and principles, but also in specifying associated methods that better ensure these principles will be appropriately applied.* This point applies as well to other proposals for instructional principles.

Instructional Design Principles require Empirical Methods to Successfully Implement

The National Research Council's volume How People Learn (Bransford, Brown, & Cocking, 1999) proposes three general instructional principles: 1) build on prior knowledge, 2) connect learning of facts and procedures with conceptual learning, and 3) support meta-cognition. These are common to many other approaches. To take the

first, for instance, the NCTM states: "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge," (NCTM, 2000, p. 16). Romberg & de Lange (2002) describe the Realistic Mathematics Education approach as recommending "making use of students informal mathematical activity to support their development of more formal strategies".

But, what prior, informal knowledge do students have? Sometimes theoretical analysis is used to predict prior knowledge under the assumption that smaller component tasks are more likely to tap prior knowledge than larger whole tasks (cf., Van Merrienboer, 1997). Such analysis focuses on mathematical content and its external forms. However, smaller tasks are not always simpler. It is not the surface form of tasks or external representations that determine how accessible they are to students. Instead, it is the internal mental representations that students acquire and use in task performance that determines what will be simple or not. Thus, to identify what prior knowledge students have and whether task X is going to be more likely than task Y to tap prior knowledge, it is not sufficient to analyze the content domain. Instead it is critical to study how students actually perform on tasks – to see student thinking as it really is, not as a content analysis might assume it to be.

Results of a Difficulty Factors Assessment indicating that "smaller is not always simpler"

Consider the three problems shown below, a story problem, word problem, and equation, all with the same underlying quantitative structure and the same solution.

Story Problem: As a waiter, Ted gets $6 per hour. One night he made $66 in tips and earned a total of $81.90. How many hours did Ted work?

Word Problem: Starting with some number, if I multiply it by 6 and then add 66, I get 81.90. What number did I start with?

Equation: x * 6 + 66 = 81.90

Which would be most difficult for high school students in a first year algebra course? Nathan & Koedinger (2000) discussed results of surveys of teachers and mathematics education researchers on a variation of this question. The survey respondents tended to predict that such story problems would be most difficult and such equations would be easiest. Typical justifications for this prediction include that the story problem requires more reading or that the way the story problem is solved is by translating to the equation.

In contrast, what we found in the two original studies (Koedinger & Nathan, in press) and in subsequent replications is that students perform best at the story and word problems (70% and 61% respectively) like these and worst at the analogous equations (42%). Clearly many students were not solving the story and word problems using equation solving. Instead they used alternative informal strategies like guess-

and-test and "unwinding", working backwards from the result, inverting operations to find the unknown starting quantity (cf., Bednarz, Kieran, & Lee, 1996). Students had difficulty in comprehending equations and, when they did succeed in comprehending, they often had difficulty in reliably executing the equation solving strategy.

This result is important within the algebra domain. It indicates that if we want to create instruction that builds on prior knowledge, we should make use of the fact that beginning algebra students have quantitative reasoning skills that can be tapped through verbal or situational contexts. Unlike many textbooks that teach equation solving prior to story problem solving, it may be better to use story problem situations and verbal descriptions first to help students understand quantitative relationships with them before moving to more abstract processing.

However, there is also a more general message. In order to apply principles like "build on prior knowledge" we cannot assume that it is obvious what prior knowledge students' posses. Instead, we need to do empirical studies to assess what prior knowledge students have and what forms of presentation best elicit this knowledge. To remind us of this message, we need to repeat the mantra: "The student is not like me".

Cognitive Tutors are Developed Using the Design Experiment Methodology

Assessment experiments, like the one above, that compare the effect of various difficulty factors on student performance are a common component in my instructional design approach. Such "Difficulty Factors Assessments" are one part of my version of the "design experiment" (Brown, 1992) approach. Design experiments involve theoretically motivated cycles of design, feedback, and redesign. While use of theory is critical to the approach, design experiments do not involve a linear process of applying theory to create innovations that are then tested. The approach is neither purely basic nor purely applied, but combines the goals of both. It is between "the hare of intuitive design and the tortoise of cumulative science". In addition to theory, we rely on intuitions of multiple team members, particularly practicing teachers to help generate design ideas. We test and prune ideas in embedded cycles at different time scales (see Table 2). Some cycles are short, like a 5-minute assessment to test a prior knowledge hypothesis or feedback from a project teacher on how a lesson went today and how might it be improved for tomorrow. Some cycles are at the curriculum unit (or chapter) level, like investigating how much a unit improved learning from pre- to post-test relative to an alternative approach. A typical "parametric" study compares different versions of a Cognitive Tutor that vary on a key dimension of interest. The longest cycles are at the full course level, comparing student end-of-course performance with that resulting from alternative courses.

Instructional Bridging Principles

I now describe four "instructional bridging" principles that are more specific, particularly to mathematics, than the more general principles from ACT-R and others

Table 2. Empirical Methods at Different Temporal Grain Sizes

1. Single point assessment of performance. Paper quiz or informal observation (minutes).
2. Lab study or class pull-out. Pre-Post lesson length comparison (day)
3. Replacement unit "parametric" study. Pre-Post unit length comparison (week/month)
4. Full course field study. Pre-Post course length comparison (semester/year)
3. Instructional Bridging Principles

described above. I have abstracted these principles from our experience developing Cognitive Tutor courses, from associated observations and experiments, and from the mathematics education and cognitive and developmental psychology literatures. In this section, I introduce the four principles and the underlying motivation for them. In the next section, I use examples from the 6th grade Cognitive Tutor course my team is developing to illustrate an on-going application of these principles.

The four "instructional bridging" principles are:

1. *Situation-Abstraction*: Bridge from concrete situational to abstract symbolic representations.
2. *Action-Generalization*: Bridge from doing with instances to explaining with generalizations.
3. *Visual-Verbal*: Integrate pictorial and verbal/symbolic representations.
4. *Conceptual-Procedural*: Integrate conceptual and procedural instruction.

These are specific variations on more general principles like "build on prior knowledge" and "connect and integrate multiple representations of knowledge".

Different kinds of evidence for these principles can be associated with the different kinds of studies in Table 2. One heuristic form of evidence (#1 in Table 2) assesses students' performance to see whether one form of activity/representation is closer to students' prior knowledge and thus easier. Another stronger form of evidence is instructional studies (#2 or #3) where an experimental comparison is made between instruction based on the principle and some other plausible form of instruction. A third less strong form of evidence is full course field studies (#4) where the experimental curriculum is consistent with the principle in question, but the comparison curriculum is not. The limitation of full course studies is that experimental and comparison curricula inevitably differ in many more ways than whether they are consistent or not with the principle in question.

Situation-Abstraction Principle: Bridge from Concrete Situational to Abstract Symbolic

The "Situation-Abstraction" principle is a consistent with one of the key features of the Cognitive Tutor Algebra course. It recommends instruction that uses more familiar concrete problem situations and descriptions as a bridge to powerful abstract symbolic forms. A key premise is that for many ideas and concepts, we can find concrete situations or verbal descriptions that use or communicate these ideas in ways that are more comprehensible and accessible to students than more abstract symbolic and conventionalized forms of mathematical expression. Of course, these conventional symbolic forms are crucial to powerful thinking and communication, particularly in math and science and so are critical goals of instruction. The key hypothesis is that robust competence with abstract symbols can be reached more effectively and efficiently by employing this Situation-Abstraction principle.

This principle is similar or even identical to recommendations of other approaches (e.g., Nunes, Schliemann, & Carraher, 1993; Collins, Brown, & Newman, 1989; Cognition and Technology Group (CTG), 1997). For instance, the Realistic Mathematics Education approach recommends "developing instruction based in experientially real contexts" (Romberg & de Lange, 2002, p. 9). This approach has been nicely implemented in the Math in Context curriculum (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997–1998).

Evidence for this principle includes of a number of full-year field studies (CTG, 1997; Koedinger et al, 1997) and some unit-level studies (Brenner, Mayer, Moseley, Brar, Duran, Reed, & Webb, 1997; Nathan, Stephens, Masarik, Alibali, & Koedinger 2002). However, all these studies involve substantial curricula that vary in other ways from the comparison curricula (e.g., use of multiple representations). The theoretical rationales for the benefits of early use of concrete or authentic situations include better connection with student prior knowledge, facilitating student motivation, facilitating memory, and facilitating transfer to authentic real world goals. We need more studies to test these different underlying rationales. My colleagues and I have emphasized using situations to connect with prior knowledge and demonstrated, as described above, that certain problem situations are easier for students to understand than corresponding symbolic problems. However, we have not "closed the loop" in a scientifically rigorous way, for instance, by showing that instruction that introduces situations before abstractions leads to better learning than instruction that introduces abstractions before situations but is otherwise the same.

While this principle is well known, it is not necessarily easy to implement effectively in the classroom as Deborah Ball (1993) noted:

> How do I (as a mathematics teacher) create experiences for my students that connect with what they now know and care about but that also transcend the present? How do I value their interests and also connect them to ideas

> and traditions growing out of centuries of mathematical exploration and invention? (p. 375)

We should not expect that all kinds of situational support are valuable in all domains. Whether and what kinds of situational supports make sense or are interesting to students depends on the domain – a point I will illustrate below with data from the 6th grade Cognitive Tutor project.

Action-Generalization Principle: Bridge from Doing with Instances to Explaining with Generalizations

The "Action-Generalization" principle is also consistent with the Cognitive Tutor Algebra course. It recommends instruction that has students engaging in problem solving (doing) with specific instances of a mathematical relationship as a bridge to explaining that relationship in general terms. For instance, we might have students solve a problem for specific numerical instances (e.g., 2 * 4 = 8 and 3 * 4 = 12) before having them explain what they are doing in a general form (e.g., x * 4; cf., Bednarz, Kieran, & Lee, 1996). Or, we might have students measure and add the angles in specific triangles before discovering the generalization and explaining it in their own words.

In the process of designing the Algebra Cognitive Tutor, we performed a parametric study in which we compared tutor versions with different orderings of questions about the problem situation (Koedinger & Anderson, 1998). One tutor version was modeled after a popular algebra textbook (Forester, 1984). Here students were asked first to define an independent variable and write an expression for the dependent quantity then asked to solve for multiple instances of the variable. Another tutor version was consistent with the Action-Generalization principle. In this version, students were first asked to solve the problem situation for a couple specific instances and then asked to write an expression for the dependent quantity. Students showed significantly greater pre-to-post learning gain in the version in which instances preceded the general expression, consistent with the Action-Generalization principle.

The Action-Generalization principle is a variation on the Realistic Mathematics Education notion of progressive formalization. Hans Freudenthal (1983) believed that "students are entitled to recapitulate in a fashion the learning process of mankind" (p. ix). This principle is also related to other recommendations for promoting student meta-cognition (e.g., Bransford et al., 1999), self-reflection, and particularly self-explanation (e.g., Chi, de Leeuw, Chiu, & Lavancher, 1994). We performed a series of parametric studies with the Geometry Cognitive Tutor (Aleven & Koedinger, 2002) comparing a textbook-like version in which students solved for measures in given figures with a self-explanation version in which students explained their problem-solving steps by indicating the relevant geometry rule. These experiments demonstrated a "less-is-more" benefit for self-explanation whereby students learn deeper,

more transferable knowledge from self-explaining even though the extra time needed for explanation means they solve fewer problems in the same instructional period. These experiments also demonstrated that such benefits can be achieved with a simple approach to explanation (referencing rules in a glossary) that is easily implemented in computer software. We are currently making progress on the more difficult task of having the software give feedback on student explanation written in their own words (Aleven, Popescu, Koedinger, 2002).

Both the algebra and geometry studies were motivated, at least in part, by results of Difficulty Factors Assessments. We had found that students were more successful at problem solving with specific instances than at explaining in general terms both in algebra (Heffernan & Koedinger, 1998) and geometry (Aleven & Koedinger, 2002). Unlike the evidence for the Situation-Abstraction principle, the evidence for the Action-Generalization principle fairly nicely "closes the loop" in the following sense. First, the two parametric studies show how a result from a single point assessment can be used to guide the application of an instructional principle to create a novel instructional design. Second, they show that the students learn more from an instructional design consistent with the Action-Generalization principle than from one that is otherwise the same, but is not consistent with the principle.

Visual-Verbal Principle: Integrate Pictorial and Verbal Representations

The Visual-Verbal principle recommends instruction that helps students integrate visual, spatial, or analog representations of an idea with verbal, sequential, or digital representations of that idea. In Cognitive Tutor Algebra, the use of multiple representations of functions, the more visual graph and table representations and the verbal-symbolic situation and equation representations, is consistent with this principle. However, it was not generally or explicitly applied. This principle is an expression of a major theme of the work of Robbie Case and colleagues (Griffin, Case, & Siegler, 1994; Kalchman, Moss, & Case, 2001). Case observed that students typically come to a new idea both with relevant visual intuitions (e.g., sense of size of quantities) and relevant verbal knowledge (e.g., knowing the counting sequence: one, two, three…). He suggested that deeper understanding and competence comes from a careful integration of this visual and verbal knowledge into a "central conceptual structure". Case and colleagues have successfully employed this principle in at least three areas, early number, rational number, and functions (Kalchman, Moss, & Case, 2001). Typical results show students in experimental conditions with poorer backgrounds (e.g., low SES 4 year olds) learning more than students in control conditions with similar backgrounds and catching up or exceeding students in control conditions with better backgrounds (e.g., high SES 4 year olds).

Another influence on this principle is the Singapore Primary Mathematics textbook series (Singapore Ministry of Education, 1999), the preface of which states:

> The main feature of the package is the use Concrete => Pictorial => Abstract approach. The pupils are provided with the necessary learning experiences beginning with the concrete and pictorial stages, followed by the abstract stage to enable them to learn mathematics meaningfully. (p. 3)

This statement and the associated materials combine (and confound) aspects of the Situation-Abstraction, Action-Explanation, and Visual-Verbal principles. However, visual "pictorial" representations are clearly emphasized.

Conceptual-Procedural Principle: Integrate Conceptual and Procedural Instruction

The Conceptual-Procedural principle recommends instruction that supports learning of both conceptual and procedural knowledge and facilitates students in integrating the two. Rittle-Johnson & Siegler (1998) define conceptual knowledge as the understanding of principles and of relations between pieces of knowledge that are needed to solve novel tasks. They define procedural knowledge as the step-by-step actions (algorithms) efficient solving of routine problem-solving tasks. Many other researchers have addressed this distinction (e.g., Hiebert, 1986; Lesh & Landau, 1983; Ma, 1999; Starr, 2000) and there has been considerable debate about which kind of knowledge comes first developmentally and which kind should be instructed first. Some advocate that students learn concepts first and should use that knowledge to generate and select procedures (e.g., Gelman & Williams, 1998; Hiebert & Wearne, 1996). Others advocate that students learn procedures first and then should extract domain concepts from that experience (e.g., Hiebert & Wearne, 1996; Siegler, 1991). It may well be that different orderings are appropriate for different mathematical ideas. More importantly, it may be better to focus on how instruction can integrate the two in an iterative process. In such an approach, increases in one type of knowledge may lead to increases in the other type, which leads to further increases in the first (Rittle-Johnson & Alibali, 1999).

While many will acknowledge the effectiveness of Cognitive Tutors for procedural knowledge acquisition, the question arises about whether they are effective in supporting student acquisition of conceptual knowledge. Our work on Cognitive Tutor Math 6 has attempted to address this question (Rittle-Johnson & Koedinger, 2001; 2002).

Using the Bridging Principles in the Design of Cognitive Tutor Math 6

Course Overview

We started work on the middle school text and software in the fall of 1999. In 2001-2002 school year the 6th grade course included 9 units of text and 36 units of software, summarized in Table 3. To illustrate the use of the principles, I will focus on the algebra and rational number strands running through the curriculum.

Table 3. Cognitive Tutor Math 6 Course Content—2001-02 School Year

Textbook Units	Tutor Units	Strands
1. Finding Patterns & Writing Rules	Algebraic Expression Generation, Expression Evaluation, Picture Algebra	Algebra
2. Grounding Place Value in Measurement	Decimal Number Line, Decimal Place Value, Decimal Addition & Subtraction	Number
3. Grounding Decimals in Area & Perimeter	Area & Perimeter with Decimals, Picture Algebra with Multiplication	Number Geometry
4. Understanding Division	Expr Gen with Div, Compatible Numbers, Simplifying Div, Decimal Div with Area	Number Algebra
5. Understanding Fractions, Equivalence, Addition & Subtraction	Fraction Concept, Proportional Reasoning, Probability, Fraction Arithmetic, Picture Algebra with Fractions	Number Data & Probability
6. Factors & Multiples	Common Factors, Common Multiples, Scaling, Proportionality with Factors & Multiples	Number
7. Fraction Multiplication & Division	Fraction Mult with Area, Picture Division, Fraction Mult & Div with Area	Number Geometry
8. Data Analysis	Simple Histogram & Central Tendency	Data
9. Properties of Angles & Triangles	Sorting & Defining, Drawing, Discovering Properties, Angle Solving, Similar Triangles	Geometry

Emphasixing the need for content-relevant data.

Instructional principles are true or useful only to the extent that the application of them is guided by content-relevant data. Our research related to the Situation-Abstraction principle provides an example. Table 4 illustrates different content areas in middle school math where we compared concrete story problem situations with abstract context-free problems. The table shows 6th graders' average percent correct on multiple

pre-test items in each category. In three of the areas, the problem situation consistently facilitates performance significantly above the abstract problem. These are decimal place value and decimal arithmetic (Rittle-Johnson & Koedinger, 2002) and fraction addition (Rittle-Johnson & Koedinger, 2001a). In data analysis (cf., Baker, Corbett, & Koedinger, 2001), the situation facilitated performance on a global interpretation task, but not on a local interpretation task. In the area of factors and multiples, the situation reduced performance.

Application of the Situation-Abstraction principle may not be effective for concepts and procedures related to factors and multiples unless situations can be found that are easier to understand than abstract problems. While one might still want to use such a problem situation as motivation for learning, given this data, it does not appear that such a situation will provide a student with easier or more direct access to understanding.

Examples from the Algebra Strand of Cognitive Tutor Math 6

Unit 1 of the 6th grade curriculum is "Finding Patterns and Writing Rules". The curriculum starts with a pre-algebra unit because it provides a good introduction to pattern finding, induction, and explaining patterns in general terms. This unit thus sets a tone for the rest of the school year where students will engage in pattern finding and rule discovery to gain deeper access into new concepts.

Pattern induction and rule writing lessons illustrating Situation-Abstraction and Action-Generalization.

Figure 1 shows a text sample from the first lesson in Unit 1, which is intended as a fun opening activity. Students engage in both a simple pattern discovery, find the minimum number of sections (pieces of cake) for N cuts (N+1), and a difficult pattern discovery, find the maximum number of sections for N cuts (left to reader!). Students achieve various levels of success from finding correct section values for small N's, through finding how the section values increase going down in the table (i.e., the "y differences"), to stating a rule for computing the sections given the cuts (i.e., the function). In our pilot classes, most 6th graders find the pattern of y differences for both minimum and maximum, and find and state the function for the minimum but not the maximum. When students do state a rule, it is usually in words ("just add 1 to the cuts") and possibly including an instance ("say the cuts is 12, then one more is the sections"). In later lessons, students work through of a series of scaffolded situations (starting with single operator addition and going to combinations of two operators) to improve their ability to find patterns and to read and write verbal rules and algebraic expressions (see Figure 2).

Rule writing is also practiced in the tutor. The Algebraic Expression Generation tutor unit (see Figure 3) is similar to early lessons in the Algebra Cognitive Tutor. Following the Action-Generalization principle and the results of the parametric study

Table 4. Comparisons of Situational and Abstract Problems in Five Content Areas

	Decimal place value	Decimal arith	Fraction addition	Data interp-global
Situation	Show 5 different ways that you can give Ben $4.07. [A place value table was provided to scaffold answers, but is not shown here]	You had $8.72. Your grandmother gave you $25 for your birthday. How much money do you have now?	Mrs. Jules bought each of her children a chocolate bar. Jarren ate 1/4 of a chocolate bar and Alicia ate 1/5 of a chocolate bar. How much of a chocolate bar did they eat altogether?	[2 scatterplots given] Do students sell more boxes of Candy Bars or Cookies as the months pass?
% Correct	61%	65%	32%	62%
Abstract	List 5 different ways to show the amount 4.07. [Place value table given.]	Add: 8.72 + 25	Add: 1/4 + 1/5	[Scatterplots given] Are there more Moops per Zog in the Left graph or the Right graph?
% Correct	20%	35%	22%	48%

	Data interpretation-local	Factors & Multiples
Situation	[Scatterplot given] When 9 players from Anchoville were present, how many pizzas got eaten?	You work at a candy store. Your boss has asked you to figure out the different ways she could package the jelly beans and chocolate eggs, and she wants to know all the possible ways. If there are 64 jelly beans and 40 chocolate eggs and she wants each package to be the same, what are the different numbers of packages you could make?
% Correct	64%	20%
Abstract	[Scatterplot given] In the Left graph, when there were 9 Pooks, how many Feeps were there?	The common factors of 64 & 40 are:
% Correct	65%	37%

To celebrate the start of sixth grade, your class is going to have a party. They are going to buy a rectangular sheet cake. The cake needs to be sliced into pieces.

Situation 1

Your job is to get the minimum number of pieces from each cut. All the cuts must be a straight line and must start and end on a side.

Complete the chart as you finish each cake. Don't be timid; draw extra rectangles if you need them.

Number of cuts	Minimum number of sections
1	2
2	
3	
.	

Below are two examples of single cut cakes that are iced. Notice that the sections do not need to be even, just be certain that the lines are straight.

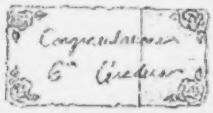

Figure 1. Text excerpt from lesson 1.1 in the 2001-02 version of Cognitive Tutor Math 6.

described above, students are first asked to solve for instances (rows 1 and 2 in Figure 3) before writing a general expression (final row). A major difference in the 6th grade tutor is the addition of the "Show Your Work" column. Having students move from instances to the algebraic expression is consistent with the notion of algebra as a generalization of arithmetic (e.g., Bednarz, Kieran, & Lee, 1996). Much to our surprise, however, we discovered that this generalization step is not the difficult one for students. In two studies reported on and referenced in Heffernan & Koedinger (1998), we found that students were not much worse at writing a general expression for story problems (49% correct) like "*X* * 2.0 – 1.25" for the problem in Figure 3 than writing an instance-based expression (53%) like "5 * 2.0 – 1.25". However, writing either kind of expression was significantly more difficult than finding a numeric solution like 8.75 for the same story problems. In other words, it is the explanation step, going from "doing with an instance" to "explaining with an instance", that is particularly difficult for students. Speaking in terms of students' learning difficulties, one might say that algebra is the "explanation of arithmetic" rather than the "generalization of arithmetic".

As indicated above, the Situation-Abstraction principle is at work in multiple places in the curriculum. Some problem situations are more authentic to real world concerns, while others, like the Cake and Marble Machine problems, are more playful. How well individual situations work either in terms of motivating students or making ideas more accessible are open questions. The process of generating problem

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Situation 3 Carol, feeling confident that she could break any code, went to the Marvelous Marble Machine. Unlike the computer code maker, the Marvelous Marble Machine uses two operators to compute its secret number.

The Marble Machine will take ordinary marbles and replace them with even more beautifu colored marbles. Of course there is a catch! You must break the code in order to keep ar marbles. If you cannot break the code, you will lose your marbles.

The following table shows how many colorful marbles you will get when you insert ordinar marbles. Help Carol break the code.

Ordinary Marbles	Colorful Marbles
2	10
3	14
6	26
9	38
125	502

2*5 2*4+2
3*4+2
6*4+2
9*4+2
125*4+2

a) How do you get from 2 ordinary to 10 colorful marbles?

b) Does this same procedure work from 3 ordinary marbles to 14 colorful marbles? (If not try another possibility for the previous question)

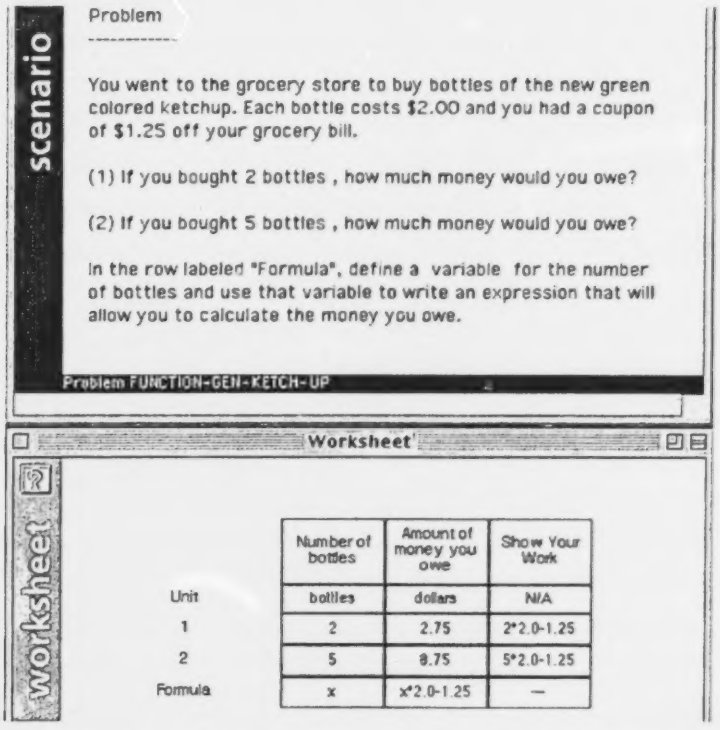
Figure 2. Text excerpt showing an example pattern finding activity with student work.

situations is primarily an art at this point, but the process of evaluating problem situations (what situations are more accessible or motivating and why) could be more of a science than it is now.

A key premise of the algebra strand is that it will be pedagogically more effective if students first learn the syntax and semantics of the algebra language by writing and reading it before learning to use this language in problem solving. While writing and reading algebraic expressions is a regular activity throughout the 6th grade curriculum, little emphasis is placed on using algebraic expressions in problem solving. Students frequently engage in algebraic problem solving, but it is supported using strategies other than equation solving. One such strategy involves the use of pictures as I describe in the next section.

Picture Algebra lessons illustrating Visual-Verbal and Conceptual-Procedural.

Figure 4 shows our initial instruction of Picture Algebra from Lesson 1.9 of the 2001-02 version of the text and Figure 5 shows an image from the associated tutor unit. This strategy is a variation of strategies used in Asian curricula (e.g., Singapore


scenario
Problem
You went to the grocery store to buy bottles of the new green colored ketchup. Each bottle costs $2.00 and you had a coupon of $1.25 off your grocery bill.
(1) If you bought 2 bottles , how much money would you owe?
(2) If you bought 5 bottles , how much money would you owe?
In the row labeled "Formula", define a variable for the number of bottles and use that variable to write an expression that will allow you to calculate the money you owe.
Problem FUNCTION-GEN-KETCH-UP
Worksheet
worksheet

	Number of bottles	Amount of money you owe	Show Your Work
Unit	bottles	dollars	N/A
1	2	2.75	2*2.0-1.25
2	5	8.75	5*2.0-1.25
Formula	x	x*2.0-1.25	—

Figure 3. Image of Algebraic Expression Generation tutor unit with "Show your work" column.

Ministry of Education, 1999). It also bears similarity with picture strategies used in Math in Context. As one point of evidence for the power of this strategy, we have observed 6th graders using it to successfully solve problems that many older students (who usually attempt an equation solving strategy) fail to solve (Koedinger & Terao, 2002). Whereas 7th graders in a sample from Bednarz & Janvier (1996, p. 120) were only 5% correct on the problem below, 6th graders in a Cognitive Tutor class were 26% correct on this problem after Picture Algebra instruction.

The use of Picture Algebra is consistent with the Visual-Verbal principle. This strategy integrates students' visual sense of quantity size with their prior verbal and symbolic skills for performing arithmetic operations. By drawing boxes of appropriate sizes, labeling, and performing associating arithmetic operations, students construct

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Situation 1

Suppose you want to buy a CD and magazine. The two cost $17 together. If the CD costs $9 more than the magazine, how much does the magazine cost?

One way to view this problem is to draw a picture to represent the objects. Be sure to label the drawings

If you remove the extra $9 from the total cost of $17, what will be left?

Figure 4. Text excerpt showing initial instruction of Picture Algebra from Lesson 1.9 of the 2001-02 version.

an integrated representation that may assist them in developing a "central conceptual structure" for algebraic reasoning.

Picture Algebra is also consistent with the Conceptual-Procedural principle. It is meant to help students acquire a conceptual understanding of key algebraic ideas. One key idea regards transformations that maintain equality, for instance, subtracting 9 from the bars on the left and the total (cf., Rittle-Johnson & Alibali, 1999). A second regards methods for recombining constraints on unknowns, for instance, seeing that the equal parts of the unknowns can be combined (analogous to transforming "$x + (x + 9)$" into "$2x + 9$"). Currently, instruction that emphasizes integration of this conceptual approach with equation solving procedures is being left for 7th and 8th grade in our curricular program. It is a good question whether or not saving the integration for later is a violation of the Conceptual-Procedural principle and, separately, whether or not earlier integration would help learning.

Examples from the Rational Number Strand of Cognitive Tutor Math 6

Rational numbers are a cornerstone of advanced mathematics and a major stumbling block for many children. Multiple units in the curriculum, especially Units 2-7, address number knowledge and rational numbers in particular.

Decimal place value and arithmetic lessons illustrating Situation-Abstraction and Conceptual-Procedural.

In text unit 2 and associated Tutor units, we targeted decimal number understanding with a particular emphasis on combining the Situation-Abstraction and the Conceptual-Procedural principles. The Conceptual-Procedural principle was employed

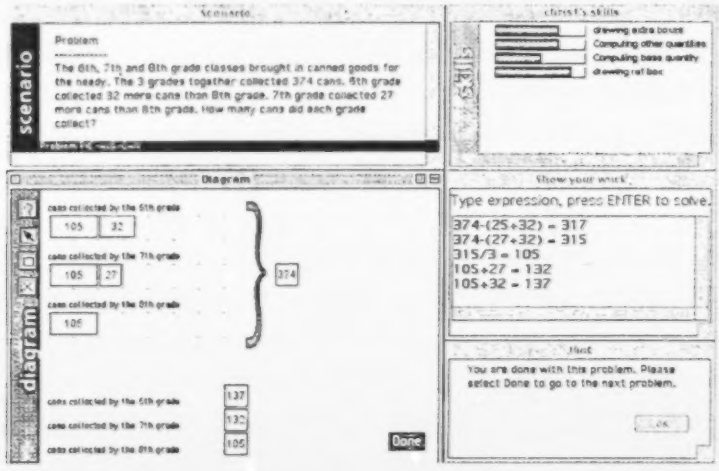
scenario
Problem
The 6th, 7th and 8th grade classes brought in canned goods for the needy. The 3 grades together collected 374 cans. 6th grade collected 32 more cans than 8th grade. 7th grade collected 27 more cans than 8th grade. How many cans did each grade collect?
skills
drawing extra boxes
Computing other quantities
Computing base quantity
drawing ref box
Diagram
diagram
cans collected by the 6th grade
105
32
cans collected by the 7th grade
105
27
cans collected by the 8th grade
105
374
137
132
105
Done
Show your work
Type expression, press ENTER to solve.
374-(25+32) = 317
374-(27+32) = 315
315/3 = 105
105+27 = 132
105+32 = 137
Hint
You are done with this problem. Please select Done to go to the next problem.
OK

Figure 5. Image of problem from Picture Algebra tutor unit. In the Diagram window, students draw boxes of appropriate size and label them (the text is given but the boxes are not). In the "Show your work" window, students must justify their answers. The tutor monitors each step and provides context-sensitive hints only as needed (by clicking on the ? icon).

through the integration of place value concepts and decimal addition and subtraction procedures. The curriculum emphasizes the alignment and adding of equivalent place values (add ones to ones, tenths to tenths, etc.) as a rationale for the procedure of aligning the decimal point. The Situation-Abstraction principle was employed through the initial use of situations involving money quantities before other non-money and abstract situations. Money contexts appear to dramatically reduce decimal alignment errors in arithmetic, as we saw in Koedinger & Nathan (in press) and in the difficulty factors comparison illustrated above. The two principles combine well here as the key abstract conceptual-procedural link, do not add different place values, has an easier to understand analog in the concrete money situation, do not add dollars and cents. In other words, add ones to ones and tenths to tenths is analogous to add dollars to dollars and dimes to dimes. Rittle-Johnson & Koedinger (2002) report on an experiment demonstrating the value of iterating between conceptual and procedural lessons, first within the familiar money situations and then more abstractly.

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Fraction concept and addition lessons illustrating Visual-Verbal and Action-Generalization.

Learning to add and subtract fractions is particularly difficult for students as indicated by the fact that incorrect procedures persist into high school and reflect basic misunderstandings of fractions (e.g., Kouba, Carpenter, & Swafford, 1989). The most common error children make is to add the numerators and denominators (e.g., 1/4 + 1/5 = 2/9), which violates basic part-whole concepts and can lead an answer that is smaller than either addend. In Rittle-Johnson & Koedinger (2001) we explored whether presenting problems with pictures or within a situational context would enhance performance and, in particular, provide a source for sense-making that would reduce this common error. Indeed, on a pre-test given to 6th graders, students performed better when given a problems with pictures (37%) or within a situation (32%; see Table 4) than when given an abstract number sentence (22%). In particular, the combine numerator and denominator error, while still common prior to instruction, was significantly lower for both pictures (38%) and situations (39%) than for the number sentence (48%). Relevant prior knowledge appears to be elicited by pictures and situations and it appears that, even without instruction, some students use such knowledge for sense making.

In addition to indicating room for improvement among 6th graders, these results suggest promise for applying the Situation-Abstraction and Visual-Verbal principles to fraction concepts and fraction arithmetic. Lesson 5.10 addressed fraction addition, but it is important to first review what ideas were addressed earlier in the unit along the developmental corridor leading to fraction addition. Students are encouraged to discover multiple ways of expressing fractions of equivalent value. Following the Visual-Verbal principle, the concept of fraction equivalence (and comparison) is supported through

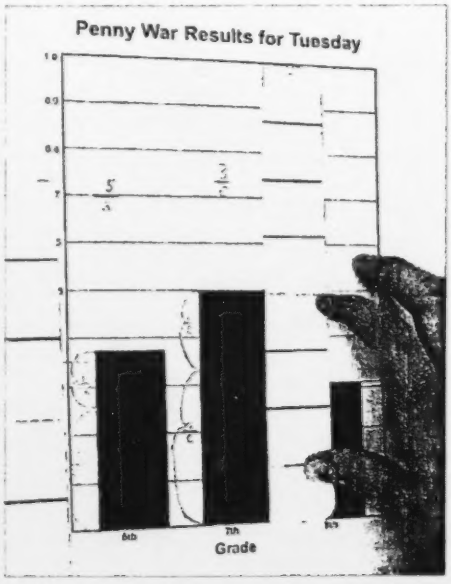

Figure 6. Student using her fraction strip to measure the result of combining the 1/3 of the weekly goal collected on Monday and the 1/6 of the weekly goal on Tuesday. She measures the result to be 1/2.

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measurement activities and pictorial representations—fraction bars, number lines, and analogous representations in the Cognitive Tutor software. As part of the measurement activities, students create a fraction ruler that is a whole unit in length and labeled with multiple fractions. This ruler is both a frequently used tool and a concrete reminder of fraction equivalence as many positions on the ruler are labeled with multiple fractions (e.g., 1/4 and 2/8).

In lesson 5.1, students begin an activity involving a fund raising effort at the middle school (see Figure 6). (Note: "Penny Wars" is the name of an annual fund-raiser at a local school, but we are changing the name in the text to "Penny Race".) The fund-raiser has a goal for the week and students record in a bar chart the fractional progress toward that goal for each grade on each day. As illustrated in Figure 6, students use their fraction ruler to measure the total of the 7th graders' penny collection on Monday, 1/3 of the goal, and on Tuesday, 1/6 of the goal. The measurement shows that the 7th graders are now 1/2 of the way to their goal.

At this point, the Action-Generalization principle is employed. Students record such measurements (actions) and instances of associated arithmetic facts (e.g., 1/3 + 1/6 = 1/2). As shown in Figure 7, students revisit these facts in lesson 5.10 and have a discussion about how to add fractions. Why is it that 1/3 + 1/6 is 1/2? The prior emphasis on fraction equivalence and its reification on the fraction ruler has the consequence that it is not a difficult leap for students to think of 1/3 as 2/6. Thus, they are in position to recognize that they already know how to add 2/6 + 1/6 (like fraction addition is a topic prior to 6th grade). Having begun to recognize a pattern, students are asked to experiment with their procedure idea on other arithmetic facts. Once their

Situation 2 – Adding Fractions with Unlike Denominators

Look back at the penny war results for Tuesday (page 7). The sixth grade collected 1/4 of its weekly goal on Monday, and 1/8 of its goal on Tuesday. The total for the week was measured at 3/8 of the total goal. The diagram and number line display the problem and the answer.

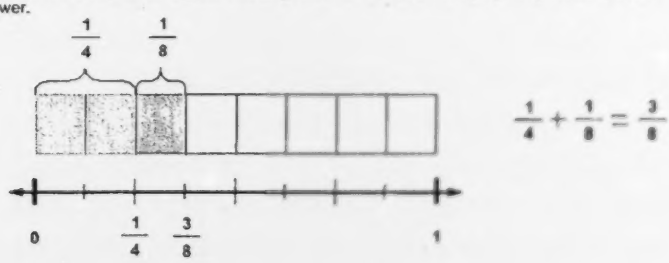

Figure 7. Revisiting the fraction addition facts determined by physical arithmetic to use as a basis for finding patterns and writing a fraction addition procedure.

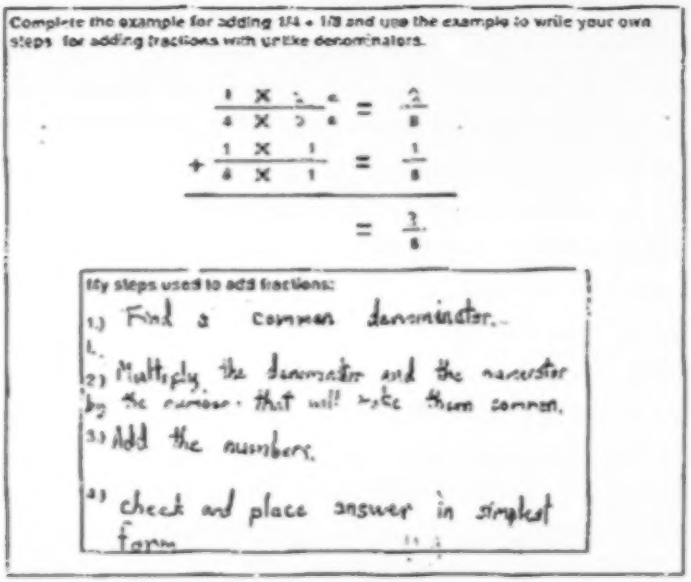
Complete the example for adding 1/4 + 1/8 and use the example to write your own steps for adding fractions with unlike denominators.
My steps used to add fractions:
1.) Find a common denominator.
2.) Multiply the denominator and the numerator by the number that will make them common.
3.) Add the numbers.
4.) check and place answer in simplest form

Figure 8. A student's procedure for addition of unlike fractions.

procedure seems to be working they write it down. Figure 8 shows one student's written procedure.

Note the progression of activities in this the application of the Action-Generalization principle. First students are "doing with instances" by using measurement to physically find fraction sums. Next, students are "doing with a generalization" by attempting to test an idea for a fraction addition procedure (which is not yet articulated) with new instances. Finally students "explain with a generalization" by writing a procedure for fraction addition in their own words. This approach is applied for other concepts in the curriculum.

An Initial Full Course Evaluation of Cognitive Tutor Math 6

We began research and development of Cognitive Tutor Math 6 in 1999 and immediately began testing in classrooms. In the 2000-2001 school year, a complete alpha version of Cognitive Tutor Math 6 was used at two Pittsburgh-area schools. To evaluate the effect of the course on raising student achievement, we contrasted end-of-course performance of students in Cognitive Tutor classes with students in schools with comparable demographics. Two types of assessments were used, a "Standard-

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ized Skills" test made up of selected test items from State and National standardized tests and a "Problem Solving and Concepts" test that we developed to assess students' problem solving abilities and conceptual understanding. The test items were selected or created to be challenging and with the goal that average performance would not be far from 50%.

Table 5 summarizes students' average percent correct on the two assessments in Cognitive Tutor and comparison classes in two school districts. All four differences between Cognitive Tutor and comparison student performance are statistically reliable.

It was not feasible to randomly assign students or teachers to tutor versus comparison classes given the logistics of agreements with the districts. Thus, there is some chance that the differences are due to other factors. However, it is encouraging that students in Cognitive Tutor classes consistently outperformed students in comparison classes on both assessments and at both schools.

Conclusions and Future Work

Cognitive Tutor courses have achieved considerable success both in terms of raising student achievement and in reaching large numbers of students. So have other research-based mathematics curricula. Nevertheless, it is critical we remain humble about our theories and beliefs about appropriate and effective mathematics instruction. There is still plenty of room for novel ideas in mathematics instruction. Moreover, there is even more room for firm evidence that existing ideas are effective in increasing student learning and do so in ways that can scale up to widespread cost-effective use. A particular gap exists between theories of general cognitive function and theories of mathematics content. We need theories and principles in this intermediate terrain that provide pedagogical leverage. Ma's (1999) work and the more general effort to characterize pedagogical content knowledge (e.g., Shulman, 1986) are extremely valuable toward filling the gap, but better still if such work makes contact with theories of general cognitive function. From the psychological side, efforts to characterize general learning and instructional principles (Bransford, et al., 1999) are also useful. However, better still if we had more specific principles and associated methods that make contact with specific pedagogical content issues.

Table 5. Results of an Initial Full Course Evaluation of Cognitive Tutor Math 6

School District	Test	Cognitive Tutor	Comparison Classes	T-test p-value
District A	Standardized	63% (N=47)	54% (N=50)	< 0.001
	Solving & Concepts	50% (N=48)	41% (N=48)	0.007
District B	Standardized	71% (N=37)	64% (N=132)	0.046
	Solving & Concepts	69% (N=38)	62% (N=116)	0.045

I have made some, I hope humble, attempts to articulate some principles within this intermediate terrain: Situation-Abstraction, Action-Generalization, Visual-Verbal, and Conceptual-Procedural. They bear considerable similarity to a number of other approaches that I am aware of and probably many more that I am not. My point is not to argue that these principles are particularly new. Nor is it to argue that I have provided firm proof for any them. Indeed, I think attempts to "prove" such principles are premature and perhaps even wrong-headed. I do suggest that we need to accumulate evidence that indicates guidelines for when and how to apply principles like these. I also suggest that such principles cannot be applied purely analytically – they require associated empirical facts, for instance, about whether particular situations or visualizations ease understanding.

This paper presented an overview of our new Cognitive Tutor Math 6 course, some samples of text and tutor activities within it, and some early evaluation data that seems to indicate that students may be learning more from this course than alternatives. I actually have not said much about the unique features of Cognitive Tutors, but have written about them elsewhere. Perhaps the most important feature is the capability to approximate the kind of moment-by-moment individualized assistance that a good human tutor can provide. The classroom teacher is a critical component in Cognitive Tutor classes, but when teachers are in the computer lab they have the support of what is, in effect, an automated teacher's aid for every student. This extra support allows them to give more individualized attention to the students who most need it (Schofield, Evans-Rhodes, & Huber, 1990; Wertheimer, 1990). Such automated support is not possible without theory-based analyses of learners and learning.

Looking forward, I would like to see mathematics educators and cognitive scientists explore the feasibility and advisability of a large program of research, on the order of the human genome project, to identify and catalog the knowledge structures or cognitive objects that underlie mathematical thinking and learning. This knowledge base would help drive new instructional innovations and provide for a rich cause theory of how, when, & why they may work. Like the human genome project, such an ambitious program would yield major applied and basic research outcomes.

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